APPENDIX 8

ELECTRE TRI

1. INTRODUCTION

The multicriteria analyses address the following problems:

(i) the choice of the best alternatives among a set of possible alternatives in order to find the best one. Alternatives may be objects, pupils, etc. or in the case of the TIRS in the Balkans project a set of identified transport infrastructure projects;

(ii) the ranking of the projects which consists in the definition of a preference order on the set of defined projects;

(iii) the sorting of the projects consists in assigning each project to one of pre-existing categories defined by norms or typical elements of a category.

In the TIRS in the Balkans project the third problem was dealt with. It belongs to the Multiple Criteria Sorting Problems family (MCSP), whose definition is: assigning a set of \( A = \{a_1, a_2, \ldots, a_j\} \) alternatives evaluated on \( n \) criteria \( g_1, g_2, \ldots, g_n \) to one of the categories which are pre-defined by some norms or corresponding to vectors of scores on particular criteria, called profiles, either separating the categories or playing the role of central reference points in the categories. The evaluation of an alternative (here a project) \( a_k \) to a specific category follows from a comparison of its evaluation on all criteria with the profiles defining the categories.

The ELECTRE TRI method was developed by the Laboratoire d'Analyse et de Modélisation de Systèmes pour l'Aide à la Décision (LAMSADE – Université Paris-Dauphine. Place du Maréchal de Lattre de Tassigny. 75775 Paris CEDEX 16. France. E-mail: mousseau@lamsade.dauphine.fr). The ELECTRE TRI 2.0 software used by Louis Berger S.A. was developed through a collaboration of two research teams: LAMSADE and the Institute of Computing Science (Poznan University of Technology, Poznan, Poland). A number of documents and references about decision aid methods and more precisely about MCSP can be found in the LAMSADE.

1.1 CRITERIA

The criteria used in the TIRS in the Balkans project have been presented and discussed in the section 7 of the report. 11 criteria were selected.

A criterion \( g \) is a real-valued function mapping from the set of alternatives (here the projects) \( A \) to \( \mathbb{R} \) the set of real numbers, so that the comparison of projects \( a \) and \( b \) may be grounded on the comparison of the two values \( g(a) \) and \( g(b) \).

Ideally, so as to enable the comparison of any pair of projects \( a \) and \( b \) in \( A \) a criterion \( g \) should be constructed such that:
\[ g(a) = g(b) \Rightarrow a I_g b \]
\[ g(a) > g(b) \Rightarrow a P_g b \]

where \( I_g \) and \( P_g \) denote the indifference and strict preference relations to criterion \( g \). In practical situations, the evaluation of projects are very often subject to imprecision, uncertainty and ill determination. Consequently a small difference of evaluation \( g(a) - g(b) \) can also imply an indifference situation. Moreover even when this difference does not seem negligible it does not always reflect a preference situation.

In order to account for the imprecision, uncertainty and ill determination of the data, it is common to use discrimination thresholds that identify the limits between situations of indifference and strict preference. Two values \( q \) and \( p \) are introduced such that:

\[ g(a) - g(b) \leq q \Rightarrow a I_g b \]
\[ q < g(a) - g(b) \leq p \Rightarrow a Q_g b \]
\[ g(a) - g(b) > p \Rightarrow a P_g b \]

where \( Q_g \) denotes the weak preference relatively to criterion \( g \). A weak preference relation is an intermediate situation that accounts for a hesitation between the situations of indifference and strict preference.

\( q \) and \( p \) are called indifference and preference thresholds respectively. Assigning values to these thresholds may be difficult. Practically reasonable values are deducted from a good understanding of the definition of the criteria and their valuation.

When the criteria are rather homogeneous and total compensation between criteria is acceptable it is frequent to build a single criterion that accounts for all pertinent aspects of the problem. In this case the evaluation of a project may be synthesized in a single value. This is usually done in the case of the complete socio-economic appraisal of a project when the net present value (NPV) or internal rate of return (IRT) aggregate the values of various benefits and costs of the project. Projects are then mutually comparable as the comparisons are made by means of comparisons of numbers. Moreover this way to proceed induces a transitive preference relation. Classical methods such as the weighted sum typically refer to this method.

The ELECTRE TRI method refuses a priori a total compensation between criteria. Incomparability is accepted so as to avoid arbitrary or fragile judgements. Transitivity of the outranking relation is not systematically imposed.

1.2 ELECTRE TRI METHOD

In the ELECTRE TRI method the assignment of a project “a” results from the comparison of “a” with the profiles defining the limits of the categories, as it is illustrated on the following graph.
For each project ELECTRE TRI validates or invalidates the statement "the project a is at least as good as the profile n". The indifference and preference thresholds q and p (see above) defined for each criterion are used for the validation.

This validation is made through the calculation of partial concordance indices (for each pair of alternatives, each criterion and each profile for this criterion). These indices are then aggregated into global concordance indices (for each pair of alternatives, all criteria taken as a whole and all profiles).

The aggregation of the calculation made for each criterion into a comprehensive level of preference is based on two conditions:

- concordance condition: the project a is at least as good as the profile n if a sufficient number of criteria are in favour of the statement;

- non-discordance condition: when the concordance condition holds, none of the criteria should oppose to the statement in a too strong way.

The procedure followed by ELECTRE TRI starts with the comparison of the project a with the highest profile, then the next one, etc. until the statement "the project a is at least as good as the profile j" is validated. The project is then assigned to the category located above the profile j.

1.3 SPECIFICATIONS FOR THE TIRS IN THE BALKANS PROJECT

223 projects were collected in the 7 study countries. 11 criteria were defined and discussed with the steering committee. Projects and criteria are presented and discussed in the section 7 of the report.

The marks given to the projects by the team of experts range from A to F. They were converted into figures from "0" (= F) to "10" (= A) with equal intervals.
Three profiles were defined corresponding to 4 categories ("I", "IIa", "IIb", "III") and adjusted in order to have approximately balanced amounts of investments in each category. The adjustment also took into accounts the distribution of marks for each criterion.

The indifference and preference thresholds are 1 and 2 respectively for all criteria and profiles.